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The Long-Run Non-Neutrality of Monetary Policy: A General Statement in a Dynamic General Equilibrium Model

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Abstract

This paper provides an explanation of the long-run non-neutrality of monetary policy in a dynamic general equilibrium (DGE) model with microfoundations. If the rate of time preference is endogenous there is no natural rate of interest. Therefore, if the central bank follows an interest rate rule this will affect the real rate of interest in financial markets and thereby the real economy. In principle, there is a negative relationship between the real rate of interest and the rate of inflation. This turns out to be nothing other than the historical forced savings effect, or the twentieth century Mundell-Tobin effect.

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Monetary non-neutrality, endogenous time preference, neo-Wicksellian model, dynamic general equilibrium model, interest rate rules

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Introduction

The idea of the long-run neutrality of changes in monetary policy is part of the DNA of the Classical approach to economic theory, going back at least to Hume in 1752 (Humphrey 1998). Of course, there have always been challenges to this position. Historically, for example, the so-called "forced savings effect" (Hayek 1932, 1939, Humphrey 1983, Smithin 2013, 2018) was often treated as a sort of exception that proves the rule to the general theoretical presumption of monetary neutrality. In the mid-twentieth century, there was considerable discussion of the analogous "Mundell-Tobin effect" named after the contributions of Mundell (1963) and Tobin (1965) which also appeared to show non-neutrality (Begg 1980, 1982, Blanchard and Fisher 1989, Smithin 1980, 2013, 2018, Turnovsky 2000, Walsh 1998). However, whether in the nineteenth century, twentieth century, or now in the twenty-first century, this sort of argument has not been well received, to say the least, by the majority of economic theorists in the mainstream of the profession.[4]

One argument that has frequently been made in recent decades is that a correct understanding of the so-called microfoundations of macroeconomics will enable the theorist to confidently rule out anything like a forced savings result. Walsh (1998, 48-9), for example, has put forward a number of arguments against some of the twentieth century demonstrations of the Mundell-Tobin effect, the most important of which is the following:

> the … behavioural relationships are *ad hoc* in the sense that they are not explicitly based on maximizing behaviour by the agents of the model. This limitation can lead to problems when we try to understand the effects of changes in the economic environment, such as changes in the rate of inflation. The effects will depend in part, on the way in which individual agents adjust, so we need to be able to predict how the demand function for money changes if the underlying time series behaviour of the inflation process were to change ... (d)oing so will ... highlight channels leading to quite different predictions than Tobin found ...

Now this sort of appeal to the microfoundations is not, in fact, a generally valid argument from either the philosophical or methodological point of view. According to King (2012, 9), for example, there are two main problems with what he unhesitatingly calls the "microfoundations

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[4] See also Kam (2000, 2005), Kam and Moshin (2006), Kam and Smithin (2012a, 2012b), Reis (2007), Smithin (2003, 2009, 2013) and Tabassum (2013).

dogma", namely, "the fallacy of composition and downward causation". Therefore;

> Since the microfoundations dogma is inconsistent with both of these principles, the dogma itself must be *false*. (Emphasis added)

Nonetheless, as suggested in the quote from Walsh, and in very many other examples in the literature, the idea that an appeal to "the" microfoundations is decisive is now almost universally accepted among the relevant peer group of academic economists. In the current intellectual environment, this situation in itself provides an extremely difficult challenge for those trying to engage in meaningful debate. Therefore, Kam (2000, 2005) took a different approach to that of King in addressing the question of monetary non-neutrality. That project was to show that non-neutrality *still* applies even in a framework which arguably has impeccable microfoundations by the standards of orthodox neoclassical economics. The purpose of the exercise was essentially to communicate with those colleagues who may be very well-versed in mathematical techniques, but not necessarily in questions of ontology and epistemology.

Kam's work, and the later work of Reis (2007),[5] was based on a modification of the well-known Sidrauski model (Sidrauski 1967) which at that time had been a staple of graduate-level textbooks for many years (Blanchard and Fisher 1989, Turnovsky 2000, Chiang and Wainwright 2005), and still is to this day. However, the canonical model in twenty-first century theoretical macroeconomics is now one version or another of either the dynamic general equilibrium (DGE) model or the dynamic *stochastic* general equilibrium (DGSE) model (DeVroey 2016, King 2012, Scarth 2014, Woodford 2010a). At this stage of the game it therefore also seems important, again for the purposes of communication, to make a more general statement about the issues in the context of a theoretical DGE model. In the Sidrauski model, the monetary policy instrument was a single monetary aggregate. In the modern DGE and DSGE frameworks the instrument is the policy rate of interest of the central bank. In turn, the difference between the DSGE and DGE frameworks is merely the extent to which stochastic calculus and the theorems of statistical probability theory play a role in the analysis. For the purposes of the present argument, dealing as it does primarily with long-run issues, consideration of a DGE model will suffice to make the

[5] Reis's paper covered much the same ground as those by Kam (2005) and Kam and Moshin (2006). However, this was not a case of successful communication. Reis's work actually appeared in the same journal as that of Kam two years earlier, but made no reference to the existing literature.

point. Woodford (2010a) had earlier employed much the same sort of strategy in his analysis of fiscal multipliers.[6]

A neo-Wicksellian DGE model with a "Representative Agent", Endogenous Money, and a Constant Rate of Time Preference

A first step is to construct a benchmark DGE model in which, by analogy to the Sidrauski model, long-run monetary neutrality *holds*. This will involve an essentially neo-Wicksellian framework with a representative agent, endogenous money, and a constant rate of time preference (Smithin 2013). The representative agent is thought of as a so-called worker-consumer and solves the following dynamic optimization problem by maximizing utility over an infinite time horizon;[7]

(1) $$Max \sum \beta^t U(C_t), \qquad U'(C_t) > 0, \; U''(C_t) < 0$$

subject to;

(2) $$W - W_{-1} = Y + r_D D - C - \delta K, \qquad 0 < \delta < 1$$

(3) $$Y = F(K), \qquad F'(K), > 0, \; F''(K) < 0$$

(4) $$W = K + D.$$

Here, W is real wealth, D is the real value of an interest-bearing financial asset denominated in the unit of account (such as interest-bearing bank deposits), K is supposedly a measure of the real capital stock,[8] Y is real GDP, r_D is the real interest rate on the financial asset, and δ is the

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[6] At just one point in the argument below, namely the specification of the optimization problem of the commercial bank, we do employ the traditional notation of stochastic calculus. Once again, this is for the purposes of communication with the target audience. However, the situation there described is not in fact a situation of probabilistic risk but of fundamental uncertainty in the Keynesian sense. Unfortunately, it is difficult to try to make this point and still retain the attention of the relevant audience. Some years ago, in his work on the "disaster point in risk theory", the late Sir John Hicks (1989, 137) faced a similar difficulty, albeit in the context of what he called "Knightian" uncertainty (after F.H. Knight) rather than the Keynesian variety. In that case also, the solution was to retain the familiar notation and assumptions, even though "I (Hicks) do not much care for them myself". In the present analysis both the main problem of the worker-consumer (see below), and the trivial problem of the central bank, are clearly non-stochastic.

[7] The seeming absurdity of these sorts of assumptions (from the point of view, that is, of the non-economist and indeed also of many heterodox economists) is actually not our main concern here. As mentioned, the objective is to conform as far as possible to the procedures typically followed in the standard literature.

[8] From the point of view of a realist approach "*per totam viam*" (Mendoza 2012) there would be yet another problem with this additional assumption. This is because the debates about capital theory in the 1950s and 1960s raised serious doubts about whether it is even possible to give any precise quantitative meaning to the notion of a physical capital stock (Harcourt 1969, Cohen and Harcourt 2003, Smithin 2018). However, and as is well-known, this problem is routinely ignored in the standard literature and has been so for the past fifty years or more.

deprecation rate. The overall problem is;

(5) $$Max \sum \beta^t \{U(C_t) + \lambda_t[F(K_t) + r_D(Wt - Kt) - C_t - \delta K_t + W_{t-1} - W_t]\};$$

noting that;

(6) $$\beta = 1/(1 + \theta).$$

The term β is usually known as the discount factor where θ stands for the rate of time preference, taken as given. In fact, the assumption of a constant rate of time preference is the precise modern equivalent of Wicksell's (1898, xxv) concept of a "natural rate" of interest. This assumption, rather than anything to do with the mathematical structure of the problem or the level of the analysis, ensures an eventual result of monetary neutrality. The first-order conditions for the solution to the optimization problem are;

(7) $$U'(C) - \lambda = 0$$

(8) $$F'(K) - r_D - \delta = 0$$

(9) $$\lambda r_D + \lambda_{+1}\beta = 0.$$

And, therefore, the dynamic system reduces to;

(10) $$U'(C)r_D = - U'(C_{+1})\beta$$

(11) $$F'(K) - \delta = r_D.$$

As shown by Kam (2000, 30-3), drawing on the literature about the "Hahn problem" (Hahn 1990) in mathematical economics from the second half of the twentieth century, the dynamic properties of this type of model with only two assets will generally involve a saddle-point. This particular specification is no exception. Therefore, *if* a plausible transversality condition can be identified, the dynamic system will converge to the following steady-state;

(12) $$r = \theta$$

(13) $$F'(K) - \delta = r_D.$$

And the overall macroeconomic equilibrium can thus be characterized as;

(14) $$r_D = \theta$$

(15) $$F'(K^N) - \delta = \theta$$

(16) $$Y^N = F(K^N)$$

Therefore, given that the central purpose of the paper is communication with the mainstream of the economics profession, we waive any further discussion of the capital theory debate.

where K^N is the equilibrium level of the capital stock, and Y^N is the equilibrium level of output.[9] Dropping the "D" subscript on the real rate of interest, the political economy of these results can be expressed even more simply as:

(17) $r = r^N \ (= \theta)$ (there is natural rate of interest)

(18) $Y = Y^N$. (there is a natural level of output)

In equilibrium, "the" real rate of interest will be at its natural level (determined by the constant rate of time preference) and output will also be at its natural level.

How to Handle Inflation in neo-Wicksellian DGE models?

As shown above, it is actually a fairly straightforward exercise to derive the real equilibrium of the benchmark neo-Wicksellian model. The results conform to what would be expected. However, and as previously discussed by Rogers (2006, 2011, 2018) and Smithin (1994, 2003, 2013), for example, there turns out to be something of a problem for all types of neo-Wicksellian theory in any attempt to include an explanation of inflation in the analysis.[10] Rogers attributes this to the fact that the class of neo-Wicksellian DGE, or DSGE, models based on the work of Woodford (1998, 2003) have no essential role for money, specifically no role for money as a medium of exchange.[11] This is a correct argument as far as it goes. Nonetheless, similar problems also arise in contexts in which, whether there is room for a medium of exchange or not, money performs other functions than that of a medium of exchange. In the Sidrauski model, for example, money enters directly as an argument in the utility function. Meanwhile, in our benchmark neo-Wicksellian model money has two essential functions. It is that which is acceptable in payment of debt, and also the means of payment function in this sense is invariably combined with that of the unit of account (Hicks 1989, Ingham 2004, 2017, Smithin 2018).

[9] The superscript, "N", carries the connotation of the "natural rates" of real interest (as in Wicksell) and output, to which the economy is always supposedly converging.

[10] As far as we know this set of issues was first raised more than thirty years ago in a paper by McCallum (1986) published some time before the model of the so-called "new consensus" was widely accepted in the literature. See also Smithin (1994, 2003) and MacKinnon and Smithin (1993). The latter paper also deals with some of the issues that have arisen in the discussion of the "neo-Fisherian" view of monetary economics. We are grateful to one of our anonymous referees who raised the question of the neo-Fisherian approach as an aside to the main discussion.

[11] Rogers (2018) actually identifies a second "Hahn problem" arising from this issue, based on some remarks by Hahn (1983) in his book on *Money and Inflation*.

Smithin (2018) has argued that these twin attributes are in fact the fundamental properties of money. Moreover, in the benchmark model money is always at least a partial store of value, as it is explicitly interest-bearing. Therefore, the difficulties in handling inflation in a neo-Wicksellian framework are more general than those which arise specifically in Woodford's approach.

To explain note that, by definition in equilibrium, the real interest rate is given by the nominal interest rate less the equilibrium inflation rate. That is;

(19) $$r = i - p.$$

But then, from equation (17), it must also be true that;

(20) $$p = i - r^N.$$

The problem in (20) is the implication that an increase in the nominal interest rate, for example, will lead to an increase in the inflation rate. But, this would not be a Wicksell-type result at all. It is entirely counter-intuitive from a Wicksellian point of view if not from an old-fashioned monetarist perspective (*cf.* MacKinnon and Smithin 1993, previously cited in footnote 10). The monetarists were always quite happy to insist that nominal interest rates would be positively correlated with inflation, with causality running from the growth rate of the money supply to inflation and then to nominal interest rates. The Wicksell-type argument, on the contrary, would be that a *lower* (not a higher) interest rate leads to *higher* inflation.

To try to reconcile these contradictions suppose that, alternatively, that we add a typical central bank reaction function to our model, the prototype of which was the famous Taylor rule (Taylor 1993). It will be immediately be noticed at this point that we have already had to postulate the existence of a second agent, in the shape of a banking system of some kind, to make the model "work". This does not, however, seriously comprise the attempt at providing microfoundations. Something of the kind is inevitable as soon as any attempt is made to introduce money into the process. Even in the original neoclassical models of money and growth descended from Sidrauski there was always, at least implicitly, a second agent present in the shape some kind of *deus ex machina* to actually issue the money (Harkness 1978, Smithin 1983). In the present case, we can therefore similarly suppose that there is a "Wicksellian Bank" (Smithin 2016c, 2018) in the system which adjusts the nominal interest rate according to the rule;

(21) $$i = i_j + \gamma p \qquad 0 < \gamma < 1$$

where the i_j are different possible values that could be chosen for the intercept in the reaction function. Then, from equation (20);

(22) $$i_j + \gamma p - p = r^N$$

and;

(23) $$p = [1/(1-\gamma)](i_j - r^N).$$

As there is only a partial adjustment coefficient ($0 < \gamma < 1$) the Wicksell-type argument still does not work. If $0 < \gamma < 1$ then $[1/(1-\gamma)] > 1$, and a setting of i_j higher than the natural rate continues to cause inflation to rise (not fall) and *vice versa*. That would still be backwards from Wicksell's point of view.

Is This Where the "Taylor Principle" Comes In?

Smithin (2013) has conjectured that the problems associated with correctly incorporating the inflation rate into neo-Wicksellian models may actually have played some role in the popularity of the disastrous policy fad known as the "Taylor principle" (as opposed to Taylor rule) in the early twenty-first century (Mankiw 2001, 2003, Davig and Leeper 2007, 2010, Woodford 2010b). This Taylor principle was the suggestion that the central bank should always to raise the nominal policy rate by *more* than one-for-one with the observed inflation rate in a pre-emptive strike against inflation. This advice turned out to be disastrous in the real world because it amounts to deliberately destabilizing real interest rates, and thereby financial markets. This is just one of many examples where mathematically-trained neoclassical economist theorists seem to have been at cross-purposes with more practically orientated market-watchers in recent decades. Nonetheless from the theorists' standpoint, and regardless of its impact on the real economy, applying the Taylor principle does actually solve some technical mathematical problems. It would give rise to a rule such as;

(24) $$i = i_j + (1 + \gamma)p. \qquad 0 < \gamma < 1$$

So that;

(25) $$i = r_j + \gamma p.$$

Next, substitute back into (20) which yields;

(26) $$r_j + \gamma p - p = r^N.$$

And finally, solving for the inflation rate, we obtain;

(27) $p = [1/(1-\gamma)](r^N - r_j)$

where now the r_j carry the connotation of the different values for the intercept that could be chosen in a "real interest rate rule" (Barrows and Smithin 2009, Smithin 1994, 2003, 2013, 2018). This is a much more Wicksellian result. The argument now is that if the real intercept in the reaction function is consistently less than the natural rate (which is effectively the constant rate of time preference) there will be inflation. After much mathematizing, the real equilibrium of the enhanced modern neo-Wicksellian model therefore finally comes down to;[12]

(28) $Y = Y^N$

(29) $p = [1/(1-\gamma)](r^N - r_j)$. $0 < \gamma < 1$

The conclusion is that in such a model the level of output Y, in equilibrium, is always at its natural value Y^N which is also supposedly the same as that which would prevail in a barter exchange economy. As far as inflation is concerned, if the "base real policy rate", as we might now call it, that is r_j, is too low relative to the natural rate r^N, there will be positive inflation and *vice versa.* These are exactly the results the neo-Wickellsian theorist would be looking for, never mind their applicability to an actual economy. Smithin's (2013, 131-2) comment on all this was as follows;

> The historically-minded reader will note that the model in … [(28)–(29)] … is only a marginal advance from position already reached by Keynes (1930, 21-44) in chapter 10 of his *Treatise on Money*.

As argued by Kam and Smithin (2019), this seems to be an unbelievably small reward for what has now been nine decades of intensive mathematical research in academia.

Endogenous Time Preference

The key move in the analysis by Kam (2000, 2005) was to endogenize the rate of time preference. It has been known at least since Uzawa (1968) that this would restore the property of monetary non-neutrality.

However, the particular specification used by Uzawa was always highly controversial (Blanchard and Fisher 1989, Kam 2000). Uzawa had assumed that time preference depends

[12] Equation (29) is the same as (27). It is included merely in order to display the complete two-equation solution system.

positively on the level of current utility which itself is an increasing function of consumption.[13] On this view, inflation raises the opportunity cost of holding real money balances and renders the initial equilibrium too costly. This increases the real interest rate and decreases the demand for real balances which increases savings and the capital stock. This specification does make the rate of time preference endogenous. However, the argument that if consumption increases the rate of time preference also increases, is not at all convincing. In effect, the very act of consumption itself is supposed to make the representative agent impatient for still more consumption. This does not seem reasonable except perhaps in the pathological case of addictive substances. Persson and Svenson (1985, 45), for example, dismiss the Uzawa specification as "... arbitrary and even counter-intuitive". Blanchard and Fischer (1989, 71) go much further, and specifically warn off budding economic theorists by stating that;

> [although the] ... specification avoids the pathological results of the constant discount rate ... the Uzawa function, with its assumption that the rate of time preference increases in instantaneous utility is not ... attractive as a description of preferences and is not recommended for general use.

Kam (2000, 2005), however, building on a suggestion by Epstein and Hynes (1983) has put forward an alternative and far more intuitively plausible method of making the rate of time preference endogenous. The idea is simply to make time preference a positive function of total real wealth rather than of consumption itself. Kam's contribution was therefore to mathematically formalize the conjecture of Epstein and Hynes. Because the wealth effect on time preference is positive this amounts to reinstating the idea that there is some sort of propensity to consume out of wealth, as well as out of income.[14] Therefore, adapting the treatment in Kam (2005, 12) for use in the present neo-Wicksellian framework let;

(30) $\theta = \theta(W)$. $\qquad \theta'(W) > 0$

The first order conditions for our optimization problem will therefore now be;

(31) $U'(C) - \lambda = 0$

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[13] Abdel-Razek (1986) has explored the stability and comparative dynamics of such a system.

[14] In fact, this was also the move made in several of the supposedly *ad-hoc* macro models criticized by Walsh and others. Endogenizing the rate of time preference simply formalizes this specification in the manner that is deemed acceptable within the microfoundations literature. See also Begg (1980, 1982), Kam (2000), and Smithin (1980, 2013).

(32) $$\lambda[F'(K) - \delta - 1) + \lambda_{+1}\beta = 0$$

(33) $$\lambda(r_D - 1) + \lambda_{+1}\beta = 0.$$

Thus the revised dynamic system (where, once again, the relevant real interest rate is r_D) turns out to be;

(34) $$F'(K) - \delta = r_D$$

(35) $$U'(C)[F'(K) - \delta] = - U'(C_{+1})\beta.$$

This will again be a saddlepoint. The steady-state of the system therefore becomes;

(36) $$F'(K) - \delta = r_D$$

(37) $$F'(K) - \delta = \theta(W).$$

There is no longer any natural rate of interest in the equilibrium of this model. The rate of time preference, the net marginal product of capital, and the real interest rate on money must all conform to the standard set by the conscious monetary policy of the central bank (Smithin 1994, 2003, 2013, 2018). Comparing the above results to those of Kam (2000, 2005) and Reis (2007), endogenizing the rate of time preference is thus seen to break the conventional result of long-run monetary neutrality, regardless of whether the monetary policy instrument is the rate of growth of the money supply itself or an interest rate. Moreover, this occurs both in the case where money is an argument in the utility function and where money is primarily a means of payment/unit of account.

A Simple Theory of Banking and the Relationship between Commercial Banks and the Central Bank

As mentioned previously one possible interpretation of the nature of the financial asset in the optimization problem is as an interest-bearing deposit in a *commercial* bank. If so, then logically speaking there has to be at least one other agent in the system in addition to the worker-consumer and the single Wicksellian bank. We now have to distinguish between two components of the banking system, namely the commercial bank and the central bank. Following Kam and Smithin (2012a), let the simplified balance sheet of the commercial bank be as follows;

Table 1: *A Simplified Commercial Bank Balance Sheet*

Assets		Liabilities	
Reserves	R	Deposits	$\$D$
Loans	L	Settlement Balances	S
	-------		--------
	$R + L$		$\$D + S$

where $\$D$ is nominal bank deposits, S stands for any negative settlement balances of the commercial bank outstanding at the central bank, R is nominal bank reserves, and L is the nominal dollar amount of loans outstanding. The optimization problem for the commercial bank is therefore;

(38) $$Max\ \Pi = i_L L - i_D D - i_0\, \sigma(S - R) - \mu L$$

where Π stands for money profit, i_L is the nominal prime lending rate, i_D is the nominal deposit rate, and i_0 is the nominal policy rate (for example, the overnight rate in Canada). Substituting in from the bank balance sheet we obtain:

(39) $$Max\ \Pi = i_L L - i_D(L + R - S) - i_0 \sigma(S - R) - \mu L.$$

As far as the σ term is concerned, if we were to use the standard notation from statistical probability theory, with all the caveats about fundamental uncertainty discussed in footnote 6 above, then we could write down something like;

(40) $$\sigma = \int_0^\infty f(x)dx.$$

In this case, σ would have the connotation of the subjective "probability" (as assessed by the commercial bank officials themselves) of the commercial bank being out of the money at the clearing house.

The expression μ might be interpreted as the average cost per dollar (or euro or yen) for making bank loans. However, another difficulty with this interpretation (which is actually in much the same sort of category as the confusion between uncertainty and risk) is that there is no precise analogue to a textbook physical production function in the field of banking (Dow and Smithin 1999). It is probably safer simply to say that μ must be high enough to cover costs and also earn a normal rate of return for the banks; given existing institutional arrangements, market structure, banking legislation, regulations, *etc.* Therefore μ is ultimately determined by these sets of conditions. Substituting in from the balance sheet the optimization problem becomes;

(41) $$\text{Max: } \Pi = i_L L - i_D(L + R - S) - i_O\sigma(S - R) - \mu L$$

where the choice variables in the problem are the volume of loans granted and the quantity of precautionary reserves banks choose to hold. First order conditions are obtained by differentiating with respect to *L* and *R*, and setting the results equal to zero;

(42) $$i_L - i_D = \mu$$

(43) $$i_D = \sigma i_O.$$

The mark-up between commercial bank lending rates and deposit rates will therefore be equal to μ, and the deposit rate in commercial banks i_D is a mark-down from the central bank's setting of the policy rate i_0. In effect, the degree of the mark-down depends on the subjective assessment of the "risk" (as this is called in neoclassical economics, as mentioned a true Keynesian would prefer to call it uncertainty) of the representative commercial bank not "keeping in step" (Keynes 1930, 23) with its putative "rivals". In the past, a similar sort of result has been called the "two-for-one" rule (*cf.* Rogers and Rymes 2000, 259). However, to get a value of exactly $\sigma = 0.5$ would require making the twin assumptions of ergodicity and a normal distribution, both of which are unlikely to hold in practice. Empirically, the value of the σ term seems to be much higher than 0.5 (typically around 0.8) but still less than unity (Collis 2016, 2018a).[15]

Combining equations (42) and (43) we can see that there is a linear relationship between the policy rate and the commercial bank lending rate, thus providing an account of how changes in the central bank policy rate are transmitted to interest rates in general. That is:

(44) $$i_L = \mu + \sigma i_0$$

Next subtract the observed inflation rate, *p*, from both sides of equation (44);

(45) $$i_L - p = \mu + \sigma r_0 - (1-\sigma)p.$$

Here the term r_0 is the inflation-adjusted real policy rate of interest, that is, the nominal policy rate adjusted for the currently observed inflation rate, or $r_0 = i_0 - p$. This gives some further

[15] In fact, this in one of the main considerations, apart from the obvious fact that we are dealing with human subjectivity rather any purely physical phenomenon, suggesting that the situation is one of genuine uncertainty rather than risk.

insight into the several discussions by Smithin (*e.g.*, 1994, 2007, 2009, 2013, 2018) about a real interest rate rule for monetary policy.[16] As a practical matter such a rule would have to involve a target for the inflation-adjusted policy rate as defined above simply because the true expected inflation rate is not known. The question now, therefore, is whether the similar inflation-adjusted real commercial bank lending rate in equation (45) can also be taken as a "proxy for" (Taylor 1993, Smithin 2018) the real lending rate itself. If so, in the absence of any other indicator on which borrowers can base their estimates, equation (45) may be re-written as;[17]

(46) $$r = \mu + \sigma r_0 - (1-\sigma)p,$$

where the r term now stands for the real interest rate actually involved in economic decision-making such as, *e.g.*, the interest rate in an investment function or in an IS curve in a macro model. Equation (46) thus shows how central bank activities can indeed have influence over the real rate of interest in the market-place, and thereby over the real economy in general. Notice, particularly, the negative theoretical relationship between inflation and real interest rates in this situation. This is nothing other than the forced saving (or Mundell-Tobin) effect as already discussed above.

A Real Interest Rate Rule for Monetary Policy?

From equations (42) and (43), we can see that it also must be the case that;

(47) $$r_D = \sigma r_0 - (1-\sigma)p.$$

This formulation raises the possibility that the central bank could actually pursue a feedback rule intending to fix the real rate of return on the financial asset (the bank deposits) held by the representative worker-consumer. To set, *e.g.,* $r_D = r'$, the central bank must follow the rule;

(48) $$r_0 = (1/\sigma)r' + [(1-\sigma)/\sigma)]p.$$

This is quite complicated in and of itself, and in practice any rule followed by the central bank is presumably going to have to be more straightforward, such as the various suggestions put forward in Smithin (1994, 2013, 2016a, 2016b, 2018). Real world central bankers will not be able to cover every contingency, and therefore their main objective should probably be not to add

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[16] Smithin (2016a, 2016b, 2018, 2019) has shown that a real rate rule will also ensure inflation stability.

[17] In long run equilibrium, of course, the actual and expected inflation rates will converge.

to the instability (unlike the case of their adherence to the Taylor principle in the early twenty-first century). However, if we are prepared to allow that in principle the central bank could follow such a rule (given an exact knowledge of the parameters, *etc*.) this would greatly simplify the theoretical model, and thereby help us to better understand some of the model's basic features (Kam and Smithin 2012b, Smithin 2013, 2018) This will therefore be the underlying assumption in what follows.

Is there a "User Cost" of Producing Capital Goods rather than Consumption Goods?

There is still one loose-end to be tied up. As we have now abandoned the Taylor Principle *per se*, the system no longer determines the inflation rate. We are back to the problems in modelling inflation that have been faced by all varieties of neo-Wicksellian theory as discussed above. This failing, however can be remedied by introducing some frictions into the problem of our representative worker-consumer. Evidently, one of the main choices that the agent faces is whether to allocate current output to investment goods (that is, to increase the capital stock) or to consumption. We can therefore reasonably suppose that there is some sort of lump-sum user cost that must be incurred when making these changes.

Let V be the nominal user cost and P the price level. According to the usual logic of profit maximization, or cost minimization, there must therefore be a further marginal condition for the representative agent as follows;

(49) $$V/P = F'(K).$$

Next suppose that nominal user costs, in what we have supposed to be necessarily a money-using system, evolve according to;

(50) $$V = V_0 P_{-1} \qquad V_0 > 1$$

Substituting (48) into (47) we obtain;

(51) $$V_0(P_{-1})/P = F'(K)$$

which implies;

(52) $$V_0/F'(K) = (1 + p).$$

This therefore suggests a positive relationship between the level of real output and inflation due to the frictions associated with switching production from consumer goods to capital goods in a monetary economy. The underlying reason for this is that the user costs do have to be paid in terms of money (*i.e.*, the bank deposits), again construed as the means of payment in a debt-based system rather than merely as facilitating barter.

Formal Results

Drawing now on each of equations (11), (37) and (52) reported above, the solution system for the complete macroeconomic model is as follows;

(53) $$F'(K) - \delta = r'$$

(54) $$\theta(K + D) = r'$$

(55) $$V_0/F'(K) = (1 + p).$$

Totally differentiating;

(56) $$F''(K)dK = dr'$$

(57) $$\theta'(W)dK + \theta'(W)dD = dr'$$

(58) $$- V_0F''(K)dK/[F'(K)]^2 = dp.$$

Therefore, the results for changes in the target real rate of interest set by the central bank are;

(59) $$dK/dr' = [1/F''(K)], \qquad < 0,$$

(60) $$dD/dr' = [1 - \theta'(W)]/\theta'(W), \qquad > 0,$$

(61) $$dp/dr' = - V_0/[F'(K)]^2, \qquad < 0.$$

In short, a tight money policy, which in this context means that the central bank sets a higher target for the real rate of interest that it would like to see being charged in the market-place, will indeed succeed in permanently reducing the inflation rate. However, the steady-state capital stock and the level of output will also permanently be reduced. Monetary policy is definitively non-neutral even in the long run. At the same time, the total real holdings of the financial asset will increase which, from a common-sense point of view, is actually not all that surprising as the relevant asset (the bank deposits) is itself assumed to be interest-bearing. In effect, society's resources are being transferred from real assets to the accumulation of financial assets. This may seem just to be the obvious consequence of deliberately increasing the rate of return to financial assets. However, as discussed above, in the past it has been extraordinarily difficult to establish the existence of this sort of effect within the framework of formal mathematical economics.

These are the same sorts of results as those found in Atesoglu and Smithin (2006, 2007), Collis (2018a) Kam (2000, 2005), Mackinnon and Smithin (1993), Moshin and Kam (2006), Reis (2007), Smithin (2003, 2009, 2013, 2018), and Tabassum (2013). They are therefore robust across a wide variety of different model specifications. From the point of view of the mainstream economist, the main thing that should be interesting about them is that in this particular case the so-called microfoundations have been provided. It is therefore no longer possible simply to dismiss the non-neutrality findings on *a priori* methodological grounds.

Conclusion

This paper has provided an explanation of the long run non-neutrality of monetary policy in the context of a DGE model with microfoundations. If the rate of time preference is endogenous there is no natural rate of interest. If the central bank pursues a real interest rate rule this will influence the real levels of both lending and deposit rates in the commercial banks, and must therefore affect the real economy *via* this route. There is a negative relationship between the inflation-adjusted real lending rate of the commercial banks and the rate of inflation itself. This is nothing other than the old forced saving effect or the twentieth century Mundell-Tobin effect.

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